Play and mathematics

“Why is it,” Milo asks, “that quite often even the things which are correct don’t seem to be right?” Why indeed? I have hideous memories of taking O level Maths. I had at last, triumphantly got one problem solved. The answer was in sheep. Then I looked at the end of the question. It said, bafflingly, “Give the answer in bags of coal.” It was like a bad dream.’

(Juster 2008: 4)

What is this chapter about?

- The importance of play and the relationship between play and mathematics in early childhood.
- Making and communicating meanings – some alternative symbolic systems in other cultures; an introduction to social semiotics and multimodality; communicating mathematical meanings.
- Why maths matters – and some of the challenges concerning teaching early ‘written’ mathematics.
- A new perspective of play that supports children’s understanding of the abstract symbolic ‘written’ language of mathematics.

This book focuses on young children from birth to 6 years in educational settings as they explore, make and communicate meanings. It provides both context and theoretical underpinnings supported by numerous examples and case studies. Above all, the book celebrates young children’s imagination and play and their ability to use their own graphical marks and symbols to make and communicate their mathematical thinking.

The importance of play

This book is both play and maths ‘rich’. It interweaves children’s graphicacy throughout (i.e. drawing, maps, writing and children’s mathematical graphics), revealing
the holistic nature of the young child’s learning and their amazing interests and abilities in this significant period of their lives.

The book challenges traditional beliefs and practices of teaching ‘written mathematics’ in early childhood, to ‘set maths free’ and empower young children. Empowerment has been shown to increase children’s confidence in mathematics and in turn to support their understandings of the abstract, symbolic written language of mathematics in ways that make personal sense.

Our work is based on poststructural and democratic perspectives that privilege young children’s power, perspectives, meanings and ‘voice’. We hope that readers will also be set free of any previous anxieties they may have had about teaching mathematics, and see that, by truly valuing and supporting children’s meaning-making and graphicy every young child can achieve in mathematics. And, since almost all of the examples originated in the children’s free, self-initiated play, we believe that this book offers some very important and positive messages about how incredibly rich with potential the combination of imagination and symbolic play can be.

Play is highly complex and for adults it can often appear confusing and difficult to understand. Teachers’ uncertainty over play may be partly due to the fact that there is no one definition of the term ‘play’. Genuine child-initiated play is spontaneous and belongs to the child: it offers a ‘non-threatening way to cope with new learning and still retain self-esteem and self-image’ (Moyles 1994: 7) and has a ‘free-flow’ quality (Bruce, 2005). Contemporary research on play is informed by cultural-historical theories and guides early childhood policies and curricula in western societies (Wood 2009). According to Vygotsky (1978), one of the features of effective play is that it is social.

Wood argues that ‘what play is, what play means on and what play does for the players is conceptualised in different ways according to the particular lenses through which researchers view play’ (2010: 12). Moyles points out that the findings from recent research on play emphasize ‘children’s voices and children’s choices . . . the research illustrating children’s intentions, motivations, meaning and modes of engagement as well as exemplifying the adults’ roles’ (2010: xiii), revealing children’s learning and development from a range of perspectives. ‘Play’ can only truly be described as play when it is the child’s own, and no amount of ‘adult-planned play’ deserves to be described in this way. There is a distinct difference between children’s free and self-initiated play (i.e. play that belongs to children, that arises from their own interests and cultural experiences and empowers them to follow their own enquiries) and ‘playful activities’, which are those that belong to the adults and often have set outcomes.

There is a long history of research in this country and internationally (e.g. Isaacs 1929; Sylva et al. 1980; Pellegrini and Smith 2005; Athey 2007) that explores children’s play from many perspectives. Additionally, play has been the subject of play scholars such as Huizinga (1950), Sutton-Smith (1997) and Caillois (2001), while Goswami (2008) has shown play to be significant for children’s cognitive development. However, Broadhead (2004, cited in Broadhead et al. 2010: 181) argues that in England our ‘play heritage was substantially eroded by the culture and climate of educational reform from the late 1980s onwards . . . to the extent that the status of play in education has been at its lowest point in the last 20 years.’
Play and mathematics

A lack of child-initiated play severely limits possibilities for children to explore and communicate their own interests and mathematical ideas. It also restricts opportunities for children to engage in the sort of dialogue that can scaffold their understanding about their graphical marks and symbols, limiting their mathematical thinking and communication. This will result in imbalanced and impoverished experiences and has implications for their understanding of ‘written’ mathematics.

Rich child-initiated play supports children’s mathematical thinking and can be considered in three (non-hierarchical) ways; all are important.

- **Symbolic play** underpinning graphicacy and the abstract, written language of mathematics.
- **Exploring and communicating mathematical thinking** through graphical representations in various play contexts.
- **‘Practical’ mathematical play**.

In their **symbolic play**, children build on their earliest awareness of relationships between objects, signs and meanings (the symbol-meaning-communication relationship). Children’s meaning-making in imagination and symbolic play underpins children’s mathematical graphics.

By **exploring and communicating mathematical thinking** through graphical representations, children follow their own interests and develop ‘meaning-full’ mathematical graphics: these mathematical explorations are not adult-planned or led.

In **practical mathematical play**, it is often the resources children use that are explicitly mathematical (e.g. number puzzles; tessellated shapes; skittles games; shop tills and money; calculators; balance scales, timers and rulers), or that offer mathematical possibilities (e.g. sand; water; collections of small artefacts). Since the mathematical potential of these resources is transparent, adults can readily identify the mathematics such as ‘volume and capacity’, ‘sorting’ or ‘counting’ in which the children are engaged.

This book focuses on the first two aspects we have described above, **symbolic play**, showing how it underpins children’s mathematical graphics, and **exploring and communicating mathematical thinking** through graphical representations in various play contexts.

**Making and communicating meanings**

Vygotsky traced the beginnings of writing in gesture and play, arguing, ‘Superficially, play bears little resemblance to the complex, mediated form of thought and volition it leads to. Only a profound internal analysis makes it possible to determine its course of change and its role in development’ (1978: 104). Since both writing and mathematical notation are symbolic written languages, we argue that it is logical to assume that the beginnings of mathematical notation also have their origins in play (Worthington 2010a).

‘Written’ mathematics is of course not the entire mathematics curriculum, but it
is a very significant aspect. Vile emphasises that "There is an inextricable connection with signs and mathematics. One might even say that mathematics consists entirely of a complex system of signs . . ." (1999: 87), and van Oers describes mathematics as 'really a matter of problem solving with symbolic tools' (2001: 63).

Sand-talk and stick-talk

While ‘written’ mathematics involves pens, paper and other writing tools and surfaces, humans have not always communicated in this way and there are peoples today who continue to use other means and media, to communicate using symbolic languages that are specific to their cultures. These symbolic languages serve to emphasize that meaning-making and communication are universal human attributes.

For example, using sand as a medium, native Australian mothers in central Australia sometimes make marks and signs with their fingers in dry sand as they tell stories to their children or make maps. Chatwin (1987: 24) recounts how one Pintupu mother tells her tale in a patter of staccato bursts and, at the same time, traces the Ancestor’s ‘footprints’ by running her first and second fingers, one after the other in a double dotted line along the ground . . . The sand drawings done for children are but sketches or ‘open versions’ of real drawings representing the real Ancestors . . . it is through the ‘sketches’ that the young learn to orient themselves to their land, its mythology and resources.

Detailing some of the conventions used in sand maps, Nash (1998) observes that sand-talk is used as ‘a type of conversation’, showing how such maps ‘combine various meaning-making [semiotic] systems in the culture, notably the spoken language, gesture and iconography, as well as song, handsign and dance’.

Jenny Green has documented sand-talk among four native Australian tribes from a linguistics perspective, looking at, among other things, the different ‘modes’ of this language. She explains: ‘A bunched, or a spread hand is used to make multiple individuated marks or dots with the tips of the fingers . . . the narrator is drawing fruits in a dish in this way’ (see Figure 1.1). ‘Rhythmic dotting made with the tips of the fingers is also used to represent particular types of motion, such as dancing’ (2009: 138).

Descriptions and examples of sand-talk show that it shares some similarities with the gestural marks made by very young children when they begin to investigate the effect of their actions on the environment and in their early drawings (Matthews 1999). Sand-talk is an example of the world’s ‘transient languages’, communicative systems that are temporary and impermanent. They include the system used by the Penan tribe of Borneo whose people, with no tradition of writing, communicate in ‘a most extraordinary dialogue . . . by means of sign sticks, branches or saplings strategically placed and decorated with symbols that convey specific messages’ in the rain forest (Wade et al. 1995: 52–3). ‘Interpreting these symbols requires both knowledge of their individual meanings and understanding of the context in which the message was left’ (1995: 53) – features that are significant for all symbolic languages.
While to our western perspective it may be difficult to appreciate sand-talk and sign sticks as symbolic languages, they are truly ‘multimodal’ communicative systems, providing a glimpse of the complexity and diversity of the languages of the world’s cultures (see e.g. Agar n.d.). They emphasize the extent to which local knowledge is contained in different symbol systems and how these relate to peoples’ lives, environments and specific cultural contexts (Harrison 2007). And, just as some indigenous Australians have communicated through sand-talk, and the Penan of Borneo use sticks to ‘talk’, young children create their own powerful ways of making and communicating meanings which we can identify, providing we are open to those ways.

Social semiotics and mathematics

Against this background there is a growing body of research into the role of representation in mathematics that is largely based on a Vygotskian, social semiotic perspective. In semiotics, ‘representation’ refers to a diverse range of symbolic tools and can include gesture, spoken words and artefacts as well as graphicy, representations on paper or other surfaces. For there to be any external representations, there must also be internal, mental representations. In effect this brings the internal, mental representations ‘out there’: through their graphical representations, children’s marks and symbols – and the meanings they embody – are mirrored back to them, revealing some of their thinking and promoting further reflections on the representations themselves.

The role of semiotics in mathematics generally has been well researched (e.g. diSessa et al. 1991; Cobb et al. 1992; Ernest 2006), although research has been limited in early childhood mathematics (e.g. Hughes 1986; Gifford 1990; Munn 1994; van Oers 2000). Pape and Tchoshanov propose that mathematical representations ‘must be thought of as tools for cognitive activity rather than products or the end result of a task’ (2001: 124).

Figure 1.1 Plural-marking hand shape
Young children’s mathematical graphics arise spontaneously though their perceived needs in play, contexts that offer realistic and personally meaningful situations in which to explore and communicate cultural knowledge and ideas. Later, when teachers and practitioners plan small group sessions they will want to ensure that the mathematics has a meaningful context and purposes that the children can understand. Vellom and Pape propose that ‘typical’ written mathematical tasks (i.e. those that involve one way of colouring-in, copying or completing) are ‘seen as end results or “products”: such products fail to engage children in connecting internal and external representations at a deep level. They promote “production” of “representations” that lack meaning and from which no relational statements can be drawn’. In contrast, ‘in more realistic learning contexts, students may make sense of complex phenomena through their efforts to construct and through the use of graphical representations of these complex systems’ (2000: 125).

However, the significance of children’s personal journeys into the written language of mathematics has until very recently rarely been acknowledged in early childhood curricula (Carruthers and Worthington 2006). We believe this is likely to have been the origin of at least some of the disinterest, disaffection and dislike of mathematics for many children throughout their school careers – and for the high percentage of adults in England for whom mathematics is ‘the elephant in the classroom’ (Boaler 2009). The result has been a widely accepted ‘can’t do’ attitude to mathematics in England (DCSF 2008a: 71).

Multimodality

Children’s symbolic play and graphicacy can be viewed as different ‘modes’ or ways of representing meanings. Gunther Kress has investigated the relationship between children’s multimodal meaning-making – with various media and artefacts, junk models, drawings and cut-outs (1997) – and the very foundations of literacy (2003).

Kress’s seminal work on multimodality is also rooted in semiotics: the foundations of multimodality can be traced back to Vygotsky’s research in the 1930s on symbolic play and children’s meanings. The growing body of research in this field is where our book begins.

In early childhood, multimodality has been studied from various perspectives including combinations such as pictures, gestures and gaze (Lancaster 2001), and the syntax of graphical marks (Lancaster 2007), reminding us of the sand- and stick-talk described earlier. However, Flewitt emphasizes that while it is now recognized that children make meanings through everything they do, in the current educational climate only certain modes are valued. She argues that it is the more easily ‘assessable’ modes of spoken and written language which are prized and that ‘the multi-modality of pre-school children’s meaning making remains undervalued and under-researched’ (2005: 209).

Why maths matters

It is widely agreed that mathematics and literacy are highly significant aspects of education. They enrich our understanding of the world, enabling us to develop and
communicate our ideas and apply them in our social lives and, in the long term, they contribute to our success in work and in society at large. ‘Success’ should not only be measured in terms of achievement in examinations, but also in terms of individuals’ interest, motivation, confidence and enjoyment – elements that are important throughout childhood and indeed throughout life. Yet for all its importance it is clear that many children continue to experience problems in their experiences of learning mathematics. Boaler (2009: 36) argues that the difficulty lies in the way in which mathematics is taught:

Students are forced into a passive relationship with their knowledge – they are taught only to follow rules and not to engage in sense-making, reasoning, or thought, acts that are critical to an effective use of mathematics. This passive approach, that characterises maths teaching in many schools, is highly ineffective.

It is in their earliest educational experiences of mathematics that children develop their personal attitudes and beliefs about the subject and about themselves as young mathematicians. Dweck and Leggett (1993, cited in Sylva and Wiltshire 1993: 32–3) showed that teachers working with children in the early years contribute to their personal beliefs about their ability to succeed, and that when meeting problems or likely failure, children respond with one of two different patterns of behaviour: either ‘helplessness’ or ‘mastery’. This also raises the vexed question of ‘setting’ for mathematics that in England is often used to group children by ‘ability’, sometimes from the age of 4, yet which ‘has no positive effects on attainment but has detrimental effects on the social and personal outcomes for children’ (Blatchford et al. 2008: 28). Moreover, Boaler (2009: 95–7) argues that:

In England we do something that is very cruel to children in mathematics classrooms that sets us apart from just about every other country in the world. We tell children from a very young age, that they are no good at maths . . . deciding that primary age children have ‘low ability’ and grouping them in a low set is damaging . . . and does nothing to raise attainment.

Challenges and attitudes

One of Boaler’s main criticisms is that children in school are often taught to memorize formulae and apply them without understanding, yet in the ‘real world’ people rarely use calculation methods they have been taught in school, using instead personal methods (see e.g. Nunes et al. 1993). ‘It is so important’, Boaler argues, ‘that schools develop flexible thinkers who can draw from a variety of mathematical principles in solving problems. The only way to create flexible thinkers is to give children experience of working in these ways’ (2009: 50).

‘Written’ mathematics has been shown to cause young children considerable difficulties (e.g. Ginsburg 1977; Hughes 1986; Dowker 2004). It appears to suffer from a restricted pedagogy more than any other subject, related in part to anxieties that some early years practitioners have about mathematics. And, since mathematics is generally viewed as a ‘hard’ subject and children in early childhood education settings are very
young, a commonly held view is that the ‘skills’ of mathematical notation (i.e. written numeral formation and written calculations) need to be taught directly.

Ernest proposed that curricula reforms depend on teachers’ beliefs and that ‘Empirical evidence suggests that teachers may interpret problems and investigations in a narrow way’ (1991: 289). Furthermore, school mathematics is often viewed as having one correct solution (Lerman 1989), frequently resulting in ‘transmission’ teaching. Therefore, as mathematics can appear to be a straitjacket, an exact science with only one answer, and play often seems messy, unruly and with no particular purpose, they are often seen as polar opposites: for most the question might be how these two apparent extremes can be reconciled. Of course mathematical play (sometimes referred to as ‘practical maths’) is of enormous value as children explore concepts that underpin number, shape, space and measure, pattern and other aspects of mathematics though sand, water, puzzles and other resources. But these experiences are not directly related to children’s developing understanding of ‘written’ mathematics, since to learn this children need to ‘write’ mathematics in ways that make personal sense.

A study of teachers’ attitudes towards mathematics showed that while they recognize the importance of mathematics for their daily lives, work and society, it was only those who felt open and confident about mathematics themselves who placed the greatest emphasis on the processes of learning the subject. In contrast, teachers who ‘feel reluctant towards mathematics . . . undervalue the aspect of process’. They have a ‘traditional’ approach to their pedagogy as van Oers (2001: 27) observed at the beginning of his study ‘. . . those teachers have the children spend most of their time just working on worksheets . . . they have to colour shapes, count or encircle objects, etc’ (Thiel 2010: 111).

The role and significance of graphicacy has been considerably undervalued in the early years, and until recently its importance has seldom been acknowledged in early childhood mathematics. This understanding has its roots in imagination and symbolic play, since this is where children begin to ‘make meanings’ (through gestures, actions, words, artefacts, models, marks or graphical signs), using them to ‘mean’ or signify something else.

In the current English Early Years Foundation Stage curriculum (EYFS) (DfES 2007a), the language area of the curriculum now known as ‘communication, language and literacy’ emphasizes its communicative role in the title, and has distinct sections entitled ‘language for communication’ and ‘language for thinking’, yet the section on mathematics (now entitled ‘problem solving, reasoning and numeracy’) fails to mention either. It is almost impossible to conceive of the ‘language’ area of any curriculum without these aspects. The same document emphasizes that building the foundations of literacy includes ‘making sense of visual and verbal signs’ (p. 39), yet this is also omitted in the mathematics section of the curriculum.

A new perspective

We have been researching children’s mathematical graphics for the past 20 years. When we began in the nursery and schools in which we taught, our intention at that time was only to develop our pedagogy to better support the children’s mathematical thinking and understanding. ‘Children’s mathematics’ – mathematics that is real and
personally meaningful and rooted in their play and social contexts – can overcome some of the problems cited here: and at its heart children’s mathematical graphics offers a new perspective in supporting their understanding of the ‘written’ language of mathematics.

Our first book (Worthington and Carruthers 2003) includes examples from the children we had taught during a period of 12 years, beginning in the early 1990s. Since then we have worked with numerous children, teachers and practitioners in other settings and schools, through focused projects in schools and local authorities, through the medium of e-learning, on professional development courses and with students at colleges and universities. Teachers who have embraced children’s mathematical graphics have generously provided all of the examples and case studies in this book, showing just what can be achieved. The observations and examples featured here have been gathered recently and demonstrate that teachers and practitioners in a wide range of settings, in different neighbourhoods, cultures and countries are supporting children’s mathematical graphics. Learning is a social practice that depends on the interest, support and understanding of sensitive early childhood educators and the learning cultures they develop. The examples and case studies in this book reveal the professional journeys teachers and practitioners have made, providing insights into their considerable conceptual shifts and celebrating young children as powerful thinkers and meaning-makers.

Rather than focus only on children’s products of mathematics, effective pedagogy (informed by research and theory, observation, reflection and understanding) can reveal the processes of children’s mathematical thinking and the semiotic potential of play and graphicacy for maths. There is no justification whatsoever for children to be taught to a narrow set of criteria that are ‘ticked off’ against a predetermined list; nor to copy, colour or complete a worksheet or something ‘mathematical’ an adult has drawn or written. Such practices fail to excite children about mathematics and undervalue their potential: they simply won’t do. Nutbrown observes that ‘children approach their learning with wide eyes and open minds, so their educators too need wide eyes and open minds to see clearly and to understand what they see . . . to see what is really happening and not what adults sometimes suppose’ (2001: 134).

As the play episodes and numerous examples in this book show, there are other, richer interpretations of ‘written’ mathematics that arise through child-initiated play in naturalistic and democratic contexts of early childhood settings, and that show how rich with potential such play (and mathematics) can be.

**Conclusion**

The practitioners who have generously shared their observations and examples with us have such ‘wide eyes and open minds’. We hope that readers will share with us our delight in and respect of the power of young children’s profound capacity to make personal meanings as they make sense of their worlds. The themes explored in this introductory chapter – play, multimodality, meaning-making and graphicacy, underpin children’s mathematical graphics and are woven throughout this book. In the next chapter we reveal something of this complexity as children explore their imagination and symbolic play.
Reflections

- Discuss your own feelings about mathematics with your colleagues: have those feelings influenced the ways in which you teach the subject?
- What mathematics do you see in children's self-initiated play? Do children choose to spontaneously ‘write’ mathematics (i.e. represent on paper or another surface) in their self-initiated play?
- With your colleagues, discuss some of the ways you have seen children represent their mathematical thinking (have you seen children do this)?

Recommended reading